***CHAPTER 11***

**RISK AND RETURN: *THE CAPITAL ASSET PRICING MODEL (CAPM)***

# Answers to Concepts Review and Critical Thinking Questions

**1.** Some of the risk in holding any asset is unique to the asset in question. By investing in a variety of assets, this unique portion of the total risk can be eliminated at little cost. On the other hand, there are some risks that affect all investments. This portion of the total risk of an asset cannot be costlessly eliminated. In other words, systematic risk can be controlled, but only by a costly reduction in expected returns.

**2.** *a.* systematic

*b.* unsystematic

*c.* both; probably mostly systematic

*d.* unsystematic

*e.* unsystematic

*f.* systematic

**3.** No to both questions. The portfolio expected return is a weighted average of the asset’s returns, so it must be less than the largest asset return and greater than the smallest asset return.

**4.** False. The variance of the individual assets is a measure of the total risk. The variance on a well-diversified portfolio is a function of systematic risk only.

**5.** Yes, the standard deviation can be less than that of every asset in the portfolio. However, βp cannot be less than the smallest beta because βp is a weighted average of the individual asset betas.

**6.** Yes. It is possible, in theory, to construct a zero beta portfolio of risky assets whose return would be equal to the risk-free rate. It is also possible to have a negative beta; the return would be less than the risk-free rate. A negative beta asset would carry a negative risk premium because of its value as a diversification instrument.

**7.** The covariance is a more appropriate measure of a security’s risk in a well-diversified portfolio because the covariance reflects the effect of the security on the variance of the portfolio. Investors are concerned with the variance of their portfolios and not the variance of the individual securities. Since covariance measures the impact of an individual security on the variance of the portfolio, covariance is the appropriate measure of risk.

**8.** If we assume that the market has not stayed constant during the past three years, then the lack in movement of Southern Co.’s stock price only indicates that the stock either has a standard deviation or a beta that is very near to zero. The large amount of movement in Texas Instruments’ (TIs’) stock price does not imply that the firm’s beta is high. Total volatility (the price fluctuation) is a function of both systematic and unsystematic risks. The beta only reflects the systematic risk. Observing the standard deviation of price movements does not indicate whether the price changes were due to systematic factors or firm specific factors. Thus, if you observe large stock price movements like that of TI, you cannot claim that the beta of the stock is high. All you know is that the total risk of TI is high.

**9.** The wide fluctuations in the price of oil stocks do not indicate that these stocks are a poor investment. If an oil stock is purchased as part of a well-diversified portfolio, only its contribution to the risk of the entire portfolio matters. This contribution is measured by systematic risk or beta. Since price fluctuations in oil stocks reflect diversifiable plus non-diversifiable risks, observing the standard deviation of price movements is not an adequate measure of the appropriateness of adding oil stocks to a portfolio.

**10.** The statement is false. If a security has a negative beta, investors would want to hold the asset to reduce the variability of their portfolios. Those assets will have expected returns that are lower than the risk-free rate. To see this, examine the Capital Asset Pricing Model:

E(*R*S) = *R*f + βS[E(*R*M) – *R*f]

If βS < 0, then the E(*R*S) < *R*f

**Solutions to Questions and Problems**

*NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.*

*Basic*

**1.** The portfolio weight of an asset is total investment in that asset divided by the total portfolio value. First, we will find the portfolio value, which is:

Total value = 145($47) + 130($86)

Total value = $17,995

The portfolio weight for each stock is:

*X*A = 145($47)/$17,995

*X*A = .3787

*X*B = 130($86)/$17,995

*X*B = .6213

**2.** The expected return of a portfolio is the sum of the weight of each asset times the expected return of each asset. The total value of the portfolio is:

Total value = $3,100 + 4,600

Total value = $7,700

So, the expected return of this portfolio is:

E(*R*p) = ($3,100/$7,700)(.098) + ($4,600/$7,700)(.127)

E(*R*p) = .1153, or 11.53%

**3.** The expected return of a portfolio is the sum of the weight of each asset times the expected return of each asset. So, the expected return of the portfolio is:

E(*R*p) = .20(.105) + .45(.161) + .35(.124)

E(*R*p) = .1369, or 13.69%

**4.** Here we are given the expected return of the portfolio and the expected return of each asset in the portfolio and are asked to find the weight of each asset. We can use the equation for the expected return of a portfolio to solve this problem. Since the total weight of a portfolio must equal 1 (100%), the weight of Stock Y must be one minus the weight of Stock X. Mathematically speaking, this means:

E(*R*p) = .112 = .127*X*X + .091(1 – *X*X)

We can now solve this equation for the weight of Stock X as:

.112 = .127*X*X  + .091 – .091*X*X

.021 = .036*X*X

*X*X = .5833

So, the dollar amount invested in Stock X is the weight of Stock X times the total portfolio value, or:

Investment in X = .5833($10,000)

Investment in X = $5,833.33

And the dollar amount invested in Stock Y is:

Investment in Y = (1 – .5833)($10,000)

Investment in Y = $4,166.67

**5.** The expected return of an asset is the sum of the probability of each state occurring times the rate of return if that state occurs. So, the expected return of each stock asset is:

E(*R*A) = .25(.06) + .55(.07) + .20(.11)

E(*R*A) = .0755, or 7.55%

E(*R*B) = .25(–.20) + .55(.13) + .20(.33)

E(*R*B) = .0875, or 8.75%

To calculate the standard deviation, we first need to calculate the variance. To find the variance, we find the squared deviations from the expected return. We then multiply each possible squared deviation by its probability, and then sum. The result is the variance. So, the variance and standard deviation of each stock are:

σA2 =.25(.06 – .0755)2 + .55(.07 – .0755)2 + .20(.11 – .0755)2

σA2 = .00031

σA = .000311/2

σA = .0177, or 1.77%

σB2 =.25(–.20 – .0875)2 + .55(.13 – .0875)2 + .20(.33 – .0875)2

σB2 = .03342

σB = .033421/2

σB = .1828, or 18.28%

**6.** The expected return of an asset is the sum of the probability of each return occurring times the probability of that return occurring. So, the expected return of the stock is:

E(*R*A) = .15(–.148) + .30(.031) + .45(.162) + .10(.348)

E(*R*A) = .0948, or 9.48%

To calculate the standard deviation, we first need to calculate the variance. To find the variance, we find the squared deviations from the expected return. We then multiply each possible squared deviation by its probability, and then add all of these up. The result is the variance. So, the variance and standard deviation are:

σ2 =.15(–.148 – .0948)2 + .30(.031 – .0948)2 + .45(.162 – .0948)2 + .10(.348 – .0948)2

σ2 = .01851

σ =.018511/2

σ = .1360, or 13.60%

**7.** The expected return of a portfolio is the sum of the weight of each asset times the expected return of each asset. So, the expected return of the portfolio is:

E(*R*p) = .15(.09) + .60(.11) + .25(.14)

E(*R*p) = .1145, or 11.45%

If we own this portfolio, we would expect to earn a return of 11.45 percent.

**8.** *a.* To find the expected return of the portfolio, we need to find the return of the portfolio in each state of the economy. This portfolio is a special case since all three assets have the same weight. To find the expected return in an equally weighted portfolio, we can sum the returns of each asset and divide by the number of assets, so the expected return of the portfolio in each state of the economy is:

Boom: *R*p = (.06 + .16 + .33)/3

*R*p = .1833, or 18.33%

Bust: *R*p = (.14 + .02 − .06)/3

*R*p = .0333, or 3.33%

To find the expected return of the portfolio, we multiply the return in each state of the economy by the probability of that state occurring, and then sum. Doing this, we find:

E(*R*p) = .75(.1833) + .25(.0333)

E(*R*p) = .1458, or 14.58%

*b.* This portfolio does not have an equal weight in each asset. We still need to find the return of the portfolio in each state of the economy. To do this, we will multiply the return of each asset by its portfolio weight and then sum the products to get the portfolio return in each state of the economy. Doing so, we get:

Boom: *R*p = .20(.06) +.20(.16) + .60(.33)

*R*p = .2420, or 24.20%

Bust: *R*p = .20(.14) +.20(.02) + .60(−.06)

*R*p = –.0040, or –.40%

And the expected return of the portfolio is:

E(*R*p) = .75(.2420) + .25(−.0040)

E(*R*p) = .1805, or 18.05%

To find the variance, we find the squared deviations from the expected return. We then multiply each possible squared deviation by its probability, and then sum. The result is the variance. So, the variance of the portfolio is:

σp2 = .75(.2420 – .1805)2 + .25(−.0040 – .1805)2

σp2 = .011347

**9.** *a.* This portfolio does not have an equal weight in each asset. We first need to find the return of the portfolio in each state of the economy. To do this, we will multiply the return of each asset by its portfolio weight and then sum the products to get the portfolio return in each state of the economy. Doing so, we get:

Boom: *R*p = .30(.24) + .40(.45) + .30(.33)

*R*p = .3510, or 35.10%

Good: *R*p = .30(.09) + .40(.10) + .30(.15)

*R*p = .1120, or 11.20%

Poor: *R*p = .30(.03) + .40(–.10) + .30(–.05)

*R*p = –.0460, or –4.60%

Bust: *R*p = .30(–.05) + .40(–.25) + .30(–.09)

*R*p = –.1420, or –14.20%

And the expected return of the portfolio is:

E(*R*p) = .20(.3510) + .35(.1120) + .40(–.0460) + .05(–.1420)

E(*R*p) = .0839, or 8.39%

*b.* To calculate the standard deviation, we first need to calculate the variance. To find the variance, we find the squared deviations from the expected return. We then multiply each possible squared deviation by its probability, and then sum. The result is the variance. So, the variance and standard deviation of the portfolio are:

σp2 = .20(.3510 – .0839)2 + .35(.1120 – .0839)2 + .40(–.0460 – .0839)2 + .05(–.1420 – .0839)2

σp2 = .02385

σp = .023851/2

σp = .1544, or 15.44%

**10.** The beta of a portfolio is the sum of the weight of each asset times the beta of each asset. So, the beta of the portfolio is:

βp = .20(.75) + .30(1.90) + .15(1.38) + .35(1.16)

βp = 1.33

**11.** The beta of a portfolio is the sum of the weight of each asset times the beta of each asset. If the portfolio is as risky as the market it must have the same beta as the market. Since the beta of the market is 1.0, we know the beta of our portfolio is 1.0. We also need to remember that the beta of the risk-free asset is zero. It has to be zero since the asset has no risk. Setting up the equation for the beta of our portfolio, we get:

βp = 1.0 = 1/3(0) + 1/3(1.61) + 1/3(βX)

Solving for the beta of Stock X, we get:

βX = 1.39

**12.** CAPM states the relationship between the risk of an asset and its expected return. CAPM is:

E(*R*i) = *R*f + [E(*R*M) – *R*f] × βi

Substituting the values we are given, we find:

E(*R*i) = .038 + (.111 – .038)(1.15)

E(*R*i) = .1220, or 12.20%

**13.** We are given the values for the CAPM except for the β of the stock. We need to substitute these values into the CAPM, and solve for the β of the stock. One important thing we need to realize is that we are given the market risk premium. The market risk premium is the expected return of the market minus the risk-free rate. We must be careful not to use this value as the expected return of the market. Using the CAPM, we find:

E(*R*i) = .104 = .038 + .07βi

βi = .94

**14.** Here we need to find the expected return of the market using the CAPM. Substituting the values given, and solving for the expected return of the market, we find:

E(*R*i) = .127 = .042 + [E(*R*M) – .042](1.20)

E(*R*M) = .1128, or 11.28%

**15.** Here we need to find the risk-free rate using the CAPM. Substituting the values given, and solving for the risk-free rate, we find:

E(*R*i) = .109 = *R*f + (.118 – *R*f)(.90)

.109 = *R*f + .1062 – .90*R*f

*R*f = .0280, or 2.80%

**16.** *a.* We have a special case where the portfolio is equally weighted, so we can sum the returns of each asset and divide by the number of assets. The expected return of the portfolio is:

E(*R*p) = (.116 + .036)/2

E(*R*p) = .0760, or 7.60%

*b.* We need to find the portfolio weights that result in a portfolio with a β of .50. We know the β of the risk-free asset is zero. We also know the weight of the risk-free asset is one minus the weight of the stock since the portfolio weights must sum to one, or 100 percent. So:

βp = .50 = *X*S(1.08) + (1 – *X*S)(0)

.50 = 1.08*X*S + 0 – 0*X*S

*X*S = .50/1.08

*X*S = .4630

And, the weight of the risk-free asset is:

*XR*f = 1 – .4630

*XR*f = .5370

*c.* We need to find the portfolio weights that result in a portfolio with an expected return of 10 percent. We also know the weight of the risk-free asset is one minus the weight of the stock since the portfolio weights must sum to one, or 100 percent. So:

E(*R*p) = .105 = .116*X*S + .036(1 – *X*S)

.105 = .116*X*S + .036 – .036*X*S

*X*S = .8625

So, the β of the portfolio will be:

βp = .8625(1.08) + (1 – .8625)(0)

βp = .932

*d.* Solving for the β of the portfolio as we did in part *b*, we find:

βp = 2.16 = *X*S(1.08) + (1 – *X*S)(0)

*X*S = 2.16/1.08

*X*S = 2

*XR*f = 1 – 2

*XR*f = –1

The portfolio is invested 200% in the stock and –100% in the risk-free asset. This represents borrowing at the risk-free rate to buy more of the stock.

**17.** First, we need to find the β of the portfolio. The β of the risk-free asset is zero, and the weight of the risk-free asset is one minus the weight of the stock, so the β of the portfolio is:

ßp = *X*W(1.2) + (1 – *X*W)(0) = 1.2*X*W

So, to find the β of the portfolio for any weight of the stock, we multiply the weight of the stock times its β.

Even though we are solving for the β and expected return of a portfolio of one stock and the risk-free asset for different portfolio weights, we are really solving for the SML. Any combination of this stock and the risk-free asset will fall on the SML. For that matter, a portfolio of any stock and the risk-free asset, or any portfolio of stocks, will fall on the SML. We know the slope of the SML line is the market risk premium, so using the CAPM and the information concerning this stock, the market risk premium is:

E(*R*W) = .123 = .04 + MRP(1.20)

MRP = .083/1.2

MRP = .0692, or 6.92%

So, now we know the CAPM equation for any stock is:

E(*R*p) = .04 + .0692βp

The slope of the SML is equal to the market risk premium, which is .0692. Using these equations to fill in the table, we get the following results:

*X*W E(*R*p) ßp

0% .0400 0

25 .0608 .300

50 .0815 .600

75 .1023 .900

100 .1230 1.200

125 .1438 1.500

150 .1645 1.800

**18.** There are two ways to correctly answer this question. We will work through both. First, we can use the CAPM. Substituting in the value we are given for each stock, we find:

E(*R*Y) = .045 + .071(1.15)

E(*R*Y) = .1267, or 12.67%

It is given in the problem that the expected return of Stock Y is 11.8 percent, but according to the CAPM, the return of the stock based on its level of risk should be 12.67 percent. This means the stock return is too low, given its level of risk. Stock Y plots below the SML and is overvalued. In other words, its price must decrease to increase the expected return to 12.67 percent.

For Stock Z, we find:

E(*R*Z) = .045 + .071(.85)

E(*R*Z) = .1054, or 10.54%

The return given for Stock Z is 10.7 percent, but, according to the CAPM, the expected return of the stock should be 10.54 percent based on its level of risk. Stock Z plots above the SML and is undervalued. In other words, its price must increase to decrease the expected return to 10.54 percent.

We can also answer this question using the reward-to-risk ratio. All assets must have the same reward-to-risk ratio, that is, every asset must have the same ratio of the asset risk premium to its beta. This follows from the linearity of the SML in Figure 11.11. The reward-to-risk ratio is the risk premium of the asset divided by its β. This is also known as the Treynor ratio or Treynor index. We are given the market risk premium, and we know the β of the market is one, so the reward-to-risk ratio for the market is .071, or 7.1 percent. Calculating the reward-to-risk ratio for Stock Y, we find:

Reward-to-risk ratio Y = (.118 – .045)/1.15 = .0635

The reward-to-risk ratio for Stock Y is too low, which means the stock plots below the SML, and the stock is overvalued. Its price must decrease until its reward-to-risk ratio is equal to the market reward-to-risk ratio. For Stock Z, we find:

Reward-to-risk ratio Z = (.107 – .045)/.85 = .0729

The reward-to-risk ratio for Stock Z is too high, which means the stock plots above the SML, and the stock is undervalued. Its price must increase until its reward-to-risk ratio is equal to the market reward-to-risk ratio.

**19.** We need to set the reward-to-risk ratios of the two assets equal to each other (see the previous problem), which is:

(.118 – *R*f)/1.15 = (.107 – *R*f)/.85

We can cross multiply to get:

.85(.118 – *R*f) = 1.15(.107 – *R*f)

Solving for the risk-free rate, we find:

.1003 – .85*R*f = .1231 – 1.15*R*f

*R*f = .0758, or 7.58%

*Intermediate*

**20.** For a portfolio that is equally invested in large-company stocks and long-term bonds:

Return = (12.1% + 6.0%)/2

Return = 9.05%

For a portfolio that is equally invested in small stocks and Treasury bills:

Return = (16.5% + 3.5%)/2

Return = 10.00%

**21.** We know that the reward-to-risk ratios for all assets must be equal (See Question 19). This can be expressed as:

[E(*R*A) – *R*f]/βA = [E(*R*B) – *R*f]/βB

The numerator of each equation is the risk premium of the asset, so:

RPA/βA = RPB/βB

We can rearrange this equation to get:

βB/βA = RPB/RPA

If the reward-to-risk ratios are the same, the ratio of the betas of the assets is equal to the ratio of the risk premiums of the assets.

**22.** *a.* We need to find the return of the portfolio in each state of the economy. To do this, we will multiply the return of each asset by its portfolio weight and then sum the products to get the portfolio return in each state of the economy. Doing so, we get:

Boom: *R*p = .4(.25) + .4(.35) + .2(.40)

*R*p = .3200, or 32.00%

Normal: *R*p = .4(.18) + .4(.13) + .2(.03)

*R*p = .1300, or 13.00%

Bust: *R*p = .4(.03) + .4(–.18) + .2(–.45)

*R*p = –.1500, or –15.00%

And the expected return of the portfolio is:

E(*R*p) = .25(.32) + .55(.130) + .20(–.150)

E(*R*p) = .1215, or 12.15%

To calculate the standard deviation, we first need to calculate the variance. To find the variance, we find the squared deviations from the expected return. We then multiply each possible squared deviation by its probability, then add all of these up. The result is the variance. So, the variance and standard deviation of the portfolio are:

σ2p = .25(.32 – .1215)2 + .55(.130 – .1215)2 + .20(–.150 – .1215)2

σ2p = .02463

σp = .024631/2

σp = .1569, or 15.69%

*b.* The risk premium is the return of a risky asset minus the risk-free rate. T-bills are often used as the risk-free rate, so:

RPi = E(*R*p) – *R*f

RPi = .1215 – .038

RPi = .0835, or 8.35%

*c.* The approximate expected real return is the expected nominal return minus the inflation rate, so:

Approximate expected real return = .1215 – .035

Approximate expected real return = .0865, or 8.65%

To find the exact real return, we will use the Fisher equation. Doing so, we get:

1 + E(*Ri*) = (1 + *h*)[1 + *e*(*ri*)]

1.1215 = (1.0350)[1 + *e*(*ri*)]

*e*(*ri*) = (1.1215/1.035) – 1

*e*(*ri*) = .0836, or 8.36%

The approximate real risk-free rate is:

Approximate expected real return = .038 – .035

Approximate expected real return = .003, or .30%

And using the Fisher effect for the exact real risk-free rate, we find:

1 + E(*Ri*) = (1 + *h*)[1 + *e*(*ri*)]

1.038 = (1.0350)[1 + *e*(*ri*)]

*e*(*ri*) = (1.038/1.035) – 1

*e*(*ri*) = .0029, or .29%

The approximate real risk premium is the approximate expected real return minus the risk-free rate, so:

Approximate expected real risk premium = .0865 – .003

Approximate expected real risk premium = .0835, or 8.35%

The exact real risk premium is the exact real return minus the risk-free rate, so:

Exact expected real risk premium = .0836 – .0029

Exact expected real risk premium = .0807, or 8.07%

**23.** We know the total portfolio value and the investment of two stocks in the portfolio, so we can find the weight of these two stocks. The weights of Stock A and Stock B are:

*X*A = $190,000/$1,000,000

*X*A = .19

*X*B = $325,000/$1,000,000

*X*B = .325

Since the portfolio is as risky as the market, the β of the portfolio must be equal to one. We also know the β of the risk-free asset is zero. We can use the equation for the β of a portfolio to find the weight of the third stock. Doing so, we find:

βp = 1.0 = *X*A(.83) + *X*B(1.19) + *X*C(1.45) + *XR*f(0)

Solving for the weight of Stock C, we find:

*X*C = .31417241

So, the dollar investment in Stock C must be:

Invest in Stock C = .31417241($1,000,000)

Invest in Stock C = $314,172.41

We also know the total portfolio weight must be one, so the weight of the risk-free asset must be one minus the asset weight we know, or:

1 = *X*A + *X*B + *X*C + *XR*f

1 = .19 + .325 + .31417241 + *XR*f

*XR*f = .17082759

So, the dollar investment in the risk-free asset must be:

Invest in risk-free asset = .17082759($1,000,000)

Invest in risk-free asset = $170,827.59

**24.** We are given the expected return of the assets in the portfolio. We also know the sum of the weights of each asset must be equal to one. Using this relationship, we can express the expected return of the portfolio as:

E(*Rp*) = .127 = *X*X(.114) + *X*Y(.0868)

.127 = *X*X(.114) + (1 – *X*X)(.0868)

.127 = .114*X*X + .0868 – .0868*X*X

.0402 = .0272*X*X

*X*X = 1.47794

And the weight of Stock Y is:

*X*Y = 1 – 1.47794

*X*Y = –.47794

The amount to invest in Stock Y is:

Investment in Stock Y = –.47794($100,000)

Investment in Stock Y = –$47,794.12

A negative portfolio weight means that you short sell the stock. If you are unfamiliar with short selling, it means you borrow a stock today and sell it. You must then purchase the stock at a later date to repay the borrowed stock. If you short sell a stock, you make a profit if the stock decreases in value.

**25.** The expected return of an asset is the sum of the probability of each state occurring times the rate of return if that state occurs. So, the expected return of each stock is:

E(*R*A) = .33(.099) + .33(.113) + .33(.059)

E(*R*A) = .0903, or 9.03%

E(*R*B) = .33(–.073) + .33(.128) + .33(.293)

E(*R*B) = .1160, or 1160%

To calculate the standard deviation, we first need to calculate the variance. To find the variance, we find the squared deviations from the expected return. We then multiply each possible squared deviation by its probability, and then sum. The result is the variance. So, the variance and standard deviation of Stock A are:

σ =.33(.099 – .0903)2 + .33(.113 – .0903)2 + .33(.059 – .0903)2

σ = .00052

σA = .000521/2

σA = .0229, or 2.29%

And the standard deviation of Stock B is:

σ =.33(–.073 – .1160)2 + .33(.128 – .1160)2 + .33(.293 – .1160)2

σ = .02240

σB = .022401/2

σB = .1497, or 14.97%

To find the covariance, we multiply each possible state times the product of each asset’s deviation from the mean in that state. The sum of these products is the covariance. So, the covariance is:

Cov(A,B) = .33(.099 – .0903)(–.073 – .1160) + .33(.113 – .0903)(.128 – .1160)

+ .33(.059 – .0903)(.293 – .1160)

Cov(A,B) = –.002304

And the correlation is:

ρA,B = Cov(A,B)/σAσB

ρA,B = –.002304/(.0229)(.1497)

ρA,B = –.6728

**26.** The expected return of an asset is the sum of the probability of each state occurring times the rate of return if that state occurs. So, the expected return of each stock is:

E(*R*J) = .30(–.050) + .55(.118) + .15(.274)

E(*R*J) = .0910, or 9.10%

E(*R*K) = .30(.029) + .55(.074) + .15(.098)

E(*R*K) = .0641, or 6.41%

To calculate the standard deviation, we first need to calculate the variance. To find the variance, we find the squared deviations from the expected return. We then multiply each possible squared deviation by its probability, and then sum. The result is the variance. So, the variance and standard deviation of Stock J are:

σ =.30(–.050 – .0910)2 + .55(.118 – .0910)2 + .15(.274 – .0910)2

σ= .01139

σJ = .011391/2

σJ = .1067, or 10.67%

And the standard deviation of Stock K is:

σ =.30(.029 – .0641)2 + .55(.074 – .0641)2 + .15(.098 – .0641)2

σ = .00060

σK = .000601/2

σK = .0244, or 2.44%

To find the covariance, we multiply each possible state times the product of each asset’s deviation from the mean in that state. The sum of these products is the covariance. So, the covariance is:

Cov(J,K) = .30(–.050 – .0910)(.029 – .0641) + .55(.118 – .0910)(.074 – .0641)

+ .15(.274 – .0910)(.098 – .0641)

Cov(J,K) = .002562

And the correlation is:

ρJ,K = Cov(J,K)/σJσK

ρJ,K = .002562/(.1067)(.0244)

ρJ,K = .9836

**27.** *a.* The expected return of the portfolio is the sum of the weight of each asset times the expected return of each asset, so:

E(*R*P) = *X*FE(*R*F) + *X*GE(*R*G)

E(*R*P) = .40(.10) + .60(.13)

E(*R*P) = .1180, or 11.80%

*b*. The variance of a portfolio of two assets can be expressed as:

σ = *X*σ + *X*σ + 2*X*F*X*G σFσGρF,G

σ = .402(.532) + .602(.792) + 2(.40)(.60)(.53)(.79)(.35)

σ = .33996

So, the standard deviation is:

σP = .339961/2

σP = .5831, or 58.31%

**28.** *a.* The expected return of the portfolio is the sum of the weight of each asset times the expected return of each asset, so:

E(*R*P) = *X*AE(*R*A) + *X*BE(*R*B)

E(*R*P) = .40(.11) + .60(.13)

E(*R*P) = .1220, or 12.20%

The variance of a portfolio of two assets can be expressed as:

σ = *X*σ + *X*σ + 2*X*A*X*BσAσBρA,B

σ = .402(.472) + .602(.812) + 2(.40)(.60)(.47)(.81)(.50)

σ = .36291

So, the standard deviation is:

σP = .362911/2

σP = .6024, or 60.24%

*b.* σ = *X*σ + *X*σ+ 2*X*A*X*BσAσBρA,B

σ = .402(.472) + .602(.812) + 2(.40)(.60)(.47)(.81)(–.50)

σ = .18017

So, the standard deviation is:

σ = .180171/2

σ = .4245, or 42.45%

*c.* As Stock A and Stock B become less correlated, or more negatively correlated, the standard deviation of the portfolio decreases.

**29.** *a*. (i) Using the equation to calculate beta, we find:

βA = (ρA,M)(σA)/σM

.85 = (ρA,M)(.31)/.20

ρA,M = .55

(ii) Using the equation to calculate beta, we find:

βB = (ρB,M)(σB)/σM

1.40 = (.50)(σB)/.20

σB = .56

(iii) Using the equation to calculate beta, we find:

βC = (ρC,M)(σC)/σM

βC = (.35)(.65)/.20

βC = 1.14

(iv) The market has a correlation of 1 with itself.

(v) The beta of the market is 1.

(vi) The risk-free asset has zero standard deviation.

(vii) The risk-free asset has zero correlation with the market portfolio.

(viii) The beta of the risk-free asset is 0.

*b.* Using the CAPM to find the expected return of the stock, we find:

*Firm A:*

E(*R*A) = *R*f + βA[E(*R*M) – *R*f]

E(*R*A) **=** .05 + .85(.12 – .05)

E(*R*A) = .1095, or 10.95%

According to the CAPM, the expected return on Firm A’s stock should be 10.95 percent. However, the expected return on Firm A’s stock given in the table is only 10 percent. Therefore, Firm A’s stock is overpriced, and you should sell it.

*Firm B:*

E(*R*B) = *R*f + βB[E(*R*M) – *R*f]

E(*R*B) **=** .05 + 1.4(.12 – .05)

E(*R*B) = .1480, or 14.80%

According to the CAPM, the expected return on Firm B’s stock should be 14.80 percent. However, the expected return on Firm B’s stock given in the table is 14 percent. Therefore, Firm B’s stock is overpriced, and you should sell it.

*Firm C:*

E(*R*C) = *R*f + βC[E(*R*M) – *R*f]

E(*R*C) **=** .05 + 1.14(.12 – .05)

E(*R*C) = .1296, or 12.96%

According to the CAPM, the expected return on Firm C’s stock should be 12.96 percent. However, the expected return on Firm C’s stock given in the table is 16 percent. Therefore, Firm C’s stock is underpriced, and you should buy it.

**30.** Because a well-diversified portfolio has no unsystematic risk, this portfolio should lie on the Capital Market Line (CML). The slope of the CML equals:

SlopeCML = [E(*R*M) – *R*f]/σM

SlopeCML = (.115 – .041)/.19

SlopeCML = .38947

*a.* The expected return on the portfolio equals:

E(*R*P) = *R*f + SlopeCML(σP)

E(*R*P) = .041 + .38947(.09)

E(*R*P) = .0761, or 7.61%

*b.* The expected return on the portfolio equals:

E(*R*P) = *R*f + SlopeCML(σP)

.20 = .041 + .38947(σP)

σP = .4082, or 40.82%

**31.** First, we can calculate the standard deviation of the market portfolio using the Capital Market Line (CML). We know that the risk-free asset has a return of 4.1 percent and a standard deviation of zero and the portfolio has an expected return of 9 percent and a standard deviation of 16 percent. These two points must lie on the Capital Market Line. The slope of the Capital Market Line equals:

SlopeCML = Rise/Run

SlopeCML = Increase in expected return/Increase in standard deviation

SlopeCML = (.09 – .041)/(.16 – 0)

SlopeCML = .3063

According to the Capital Market Line:

E(*R*I) = *R*f + SlopeCML(σI)

Since we know the expected return on the market portfolio, the risk-free rate, and the slope of the Capital Market Line, we can solve for the standard deviation of the market portfolio which is:

E(*R*M) = *R*f + SlopeCML(σM)

.11 = .041 + (.3063)(σM)

σM = (.11 – .041)/.3063

σM = .2253, or 22.53%

Next, we can use the standard deviation of the market portfolio to solve for the beta of a security using the beta equation. Doing so, we find the beta of the security is:

βI = (ρI,M)(σI)/σM

βI = (.38)(.60)/.2253

βI = 1.01

Now we can use the beta of the security in the CAPM to find its expected return, which is:

E(*R*I) = *R*f + βI[E(*R*M) – *R*f]

E(*R*I) **=** .041 + 1.01(.11 – .041)

E(*R*I) = .1108, or 11.08%

**32.** First, we need to find the standard deviation of the market and the portfolio, which are:

σM = .03911/2

σM = .1977, or 19.77%

σZ = .34071/2

σZ = .5837, or 58.37%

Now we can use the equation for beta to find the beta of the portfolio, which is:

βZ = (ρZ,M)(σZ)/σM

βZ = (.31)(.5837)/.1977

βZ = .92

Now, we can use the CAPM to find the expected return of the portfolio, which is:

E(*R*Z) = *R*f + βZ[E(*R*M) – *R*f]

E(*R*Z) **=** .044 + .92(.109 – .044)

E(*R*Z) = .1035, or 10.35%

*Challenge*

**33.** The amount of systematic risk is measured by the β of an asset. Since we know the market risk premium and the risk-free rate, if we know the expected return of the asset we can use the CAPM to solve for the β of the asset. The expected return of Stock I is:

E(*R*I) = .15(.12) + .55(.17) + .30(.06) = .1295, or 12.95%

Using the CAPM to find the β of Stock I, we find:

.1295 = .04 + .075βI

βI = 1.19

The total risk of an asset is measured by its standard deviation, so we need to calculate the standard deviation of Stock I. Beginning with the calculation of the stock’s variance, we find:

σI2 = .15(.12 – .1295)2 + .55(.17 – .1295)2 + .30(.06 – .1295)2

σI2 = .00236

σI = .002361/2 = .0486, or 4.86%

Using the same procedure for Stock II, we find the expected return to be:

E(*R*II) = .15(–.35) + .55(.12) + .30(.33) = .1125, or 11.25%

Using the CAPM to find the β of Stock II, we find:

.1125 = .04 + .075βII

βII = .97

And the standard deviation of Stock II is:

σII2 = .15(–.35 – .1125)2 + .55(.12 – .1125)2 + .30(.33 – .1125)2

σII2 = .04631

σII = .046311/2 = .2152, or 21.52%

Although Stock II has more total risk than I, it has much less systematic risk, since its beta is much smaller than I’s. Thus, I has more systematic risk, and II has more unsystematic and more total risk. Since unsystematic risk can be diversified away, I is actually the “riskier” stock despite the lack of volatility in its returns. Stock I will have a higher risk premium and a greater expected return.

**34.** Here we have the expected return and beta for two assets. We can express the returns of the two assets using CAPM. If the CAPM is true, then the security market line holds as well, which means all assets have the same risk premium. Setting the reward-to-risk ratios of the assets equal to each other and solving for the risk-free rate, we find:

(.1328 – *R*f)/1.25 = (.1012 – *R*f)/.85

.85(.1328 – *R*f) = 1.25(.1012 – *R*f)

.11288 – .85*R*f = .12650 – 1.25*R*f

.40*R*f = .01362

*R*f = .0341, or 3.41%

Using CAPM to find the expected return on the market with both stocks, we find:

.1328 = .0341 + 1.25(*R*M – .0341) .1012 = .0341 + .85(*R*M – .0341)

*R*M = .1131, or 11.31% *R*M = .1131, or 11.31%

**35.** *a.*The expected return of an asset is the sum of the probability of each state occurring times the rate of return if that state occurs. To calculate the standard deviation, we first need to calculate the variance. To find the variance, we find the squared deviations from the expected return. We then multiply each possible squared deviation by its probability, and then sum. The result is the variance. So, the expected return and standard deviation of each stock are:

*Asset 1:*

E(*R*1) = .15(.20) + .35(.15) + .35(.10) + .15(.05) = .1250, or 12.50%

σ =.15(.20 – .1250)2 + .35(.15 – .1250)2 + .35(.10 – .1250)2 + .15(.05 – .1250)2 = .00213

σ1 = (.00213)1/2 = .0461, or 4.61%

*Asset 2:*

E(*R*2) = .15(.20) + .35(.10) + .35(.15) + .15(.05) = .1250, or 12.50%

σ =.15(.20 – .1250)2 + .35(.10 – .1250)2 + .35(.15 – .1250)2 + .15(.05 – .1250)2 = .00213

σ2 = (.00213)1/2 = .0461, or 4.61%

*Asset 3:*

E(*R*3) = .15(.05) + .35(.10) + .35(.15) + .15(.20) = .1250, or 12.50%

σ =.15(.05 – .1250)2 + .35(.10 – .1250)2 + .35(.15 – .1250)2 + .15(.20 – .1250)2 = .00213

σ3 = (.00213)1/2 = .0461, or 4.61%

*b.* To find the covariance, we multiply each possible state times the product of each asset’s deviation from the mean in that state. The sum of these products is the covariance. The correlation is the covariance divided by the product of the two standard deviations. So, the covariance and correlation between each possible set of assets are:

*Asset 1 and Asset 2:*

Cov(1,2) = .15(.20 – .1250)(.20 – .1250) + .35(.15 – .1250)(.10 – .1250)

+ .35(.10 – .1250)(.15 – .1250) + .15(.05 – .1250)(.05 – .1250)

Cov(1,2) = .00125

ρ1,2 = Cov(1,2)/σ1 σ2

ρ1,2 = .00125/(.0461)(.0461)

ρ1,2 = .5882

*Asset 1 and Asset 3:*

Cov(1,3) = .15(.20 – .1250)(.05 – .1250) + .35(.15 – .1250)(.10 – .1250)

+ .35(.10 – .1250)(.15 – .1250) + .15(.05 – .1250)(.20 – .1250)

Cov(1,3) = –.002125

ρ1,3 = Cov(1,3)/σ1 σ3

ρ1,3 = –.002125/(.0461)(.0461)

ρ1,3 = –1

*Asset 2 and Asset 3:*

Cov(2,3) = .15(.20 – .1250)(.05 – .1250) + .35(.10 – .1250)(.10 – .1250)

+ .35(.15 – .1250)(.15 – .1250) + .15(.05 – .1250)(.20 – .1250)

Cov(2,3) = –.00125

ρ2,3 = Cov(2,3)/σ2 σ3

ρ2,3 = –.00125/(.0461)(.0461)

ρ2,3 = –.5882

*c.* The expected return of the portfolio is the sum of the weight of each asset times the expected return of each asset, so, for a portfolio of Asset 1 and Asset 2:

E(*R*P) = *X*1E(*R*1) + *X*2E(*R*2)

E(*R*P) = .50(.1250) + .50(.1250)

E(*R*P) = .1250, or 12.50%

The variance of a portfolio of two assets can be expressed as:

σ = *X*σ + *X*σ+ 2*X*1*X*2σ1σ2ρ1,2

σ = .502(.04612) + .502(.04612) + 2(.50)(.50)(.0461)(.0461)(.5882)

σ = .001688

And the standard deviation of the portfolio is:

σP = (.001688)1/2

σP = .0411, or 4.11%

*d.* The expected return of the portfolio is the sum of the weight of each asset times the expected return of each asset, so, for a portfolio of Asset 1 and Asset 3:

E(*R*P) = *X*1E(*R*1) + *X*3E(*R*3)

E(*R*P) = .50(.1250) + .50(.1250)

E(*R*P) = .1250, or 12.50%

The variance of a portfolio of two assets can be expressed as:

σ = *X*σ + *X*σ+ 2*X*1*X*3σ1σ3ρ1,3

σ = .502(.04612) + .502(.04612) + 2(.50)(.50)(.0461)(.0461)(–1)

σ = .000000

Since the variance is zero, the standard deviation is also zero.

*e.* The expected return of the portfolio is the sum of the weight of each asset times the expected return of each asset, so, for a portfolio of Asset 2 and Asset 3:

E(*R*P) = *X*2E(*R*2) + *X*3E(*R*3)

E(*R*P) = .50(.1250) + .50(.1250)

E(*R*P) = .1250, or 12.50%

The variance of a portfolio of two assets can be expressed as:

σ = *X*σ + *X*σ+ 2*X*2*X*3σ2σ3ρ2,3

σ = .502(.04612) + .502(.04612) + 2(.50)(.50)(.0461)(.0461)(–.5882)

σ = .000438

And the standard deviation of the portfolio is:

σP = (.000438)1/2

σP = .0209, or 2.09%

*f.* As long as the correlation between the returns on two securities is below 1, there is a benefit to diversification. A portfolio with negatively correlated securities can achieve greater risk reduction than a portfolio with positively correlated securities, holding the expected return on each stock constant. Applying proper weights on perfectly negatively correlated securities can reduce portfolio variance to 0.

**36.** *a.* The expected return of an asset is the sum of the probability of each state occurring times the rate of return if that state occurs. So, the expected return of each stock is:

E(*R*A) = .15(–.13) + .60(.12) + .25(.34)

E(*R*A) = .1375, or 13.75%

E(*R*B) = .15(–.11) + .60(.10) + .25(.31)

E(*R*B) = .1210, or 12.10%

*b.* We can use the expected returns we calculated to find the slope of the Security Market Line. We know that the beta of Stock Ais .25 greater than the beta of Stock B. Therefore, as beta increases by .25, the expected return on a security increases by .0165 (= .1375 – .1210). The slope of the security market line (SML) equals:

SlopeSML = Rise/Run

SlopeSML = Increase in expected return/Increase in beta

SlopeSML = (.1375 – .1210)/.25

SlopeSML = .0660, or 6.60%

Since the market’s beta is 1 and the risk-free rate has a beta of zero, the slope of the Security Market Line equals the expected market risk premium. So, the expected market risk premium must be 6.6 percent.

We could also solve this problem using CAPM. The equations for the expected returns of the two stocks are:

.1375 = *R*f + (βB + .25)(MRP)

.1210 = *R*f + βB(MRP)

We can rewrite the CAPM equation for Stock A as:

.165 = *R*f + βB(MRP) + .25(MRP)

Subtracting the CAPM equation for Stock B from this equation yields:

.0165 = .25MRP

MRP = .0660, or 6.60%

which is the same answer as our previous result.

**37.** *a.* A typical, risk-averse investor seeks high returns and low risks. For a risk-averse investor holding a well-diversified portfolio, beta is the appropriate measure of the risk of an individual security. To assess the two stocks, we need to find the expected return and beta of each of the two securities.

*Stock A:*

Since Stock A pays no dividends, the return on Stock A is: (P1 – P0)/P0. So, the return for each state of the economy is:

*R*Recession = ($64 – 75)/$75 = –.147, or –14.70%

*R*Normal = ($87 – 75)/$75 = .160, or 16.00%

*R*Expanding = ($97 – 75)/$75 = .293, or 29.30%

The expected return of an asset is the sum of the probability of each return occurring times the probability of that return occurring. So, the expected return of the stock is:

E(*R*A) = .20(–.147) + .60(.160) + .20(.293) = .1253, or 12.53%

And the variance of the stock is:

σ = .20(–.147 – .1253)2 + .60(.160 – .1253)2 + .20(.293 – .1253)2

σ = .0212

Which means the standard deviation is:

σA = .02121/2

σA = .1455, or 14.55%

Now we can calculate the stock’s beta, which is:

βA = (ρA,M)(σA)/σM

βA = (.70)(.1455)/.18

βA = .566

For Stock B, we can directly calculate the beta from the information provided. So, the beta for Stock B is:

*Stock B:*

βB = (ρB,M)(σB)/σM

βB = (.24)(.34)/.18

βB = .453

The expected return on Stock B is higher than the expected return on Stock A. The risk of Stock B, as measured by its beta, is lower than the risk of Stock A. Thus, a typical risk-averse investor holding a well-diversified portfolio will prefer Stock B. Note, this situation implies that at least one of the stocks is mispriced since the higher risk (beta) stock has a lower return than the lower risk (beta) stock.

*b.* The expected return of the portfolio is the sum of the weight of each asset times the expected return of each asset, so:

E(*R*P) = *X*AE(*R*A) + *X*BE(*R*B)

E(*R*P) = .70(.1253) + .30(.14)

E(*R*P) = .1297, or 12.97%

To find the standard deviation of the portfolio, we first need to calculate the variance. The variance of the portfolio is:

σ = *X*σ + *X*σ + 2*X*A*X*BσAσBρA,B

σ = (.70)2(.1455)2 + (.30)2(.34)2 + 2(.70)(.30)(.1455)(.34)(.36)

σ = .02825

And the standard deviation of the portfolio is:

σP = .028251/2

σP = .1681, or 16.81%

*c.* The beta of a portfolio is the weighted average of the betas of its individual securities. So the beta of the portfolio is:

βP = .70(.566) + .30(.453)

βP = .532

**38.** *a.* The variance of a portfolio of two assets equals:

σ = *X*σ + *X*σ + 2*X*A*X*BCov(A,B)

Since the weights of the assets must sum to one, we can write the variance of the portfolio as:

σ = *X*σ + (1 – *X*A)2σ + 2*X*A(1 – *X*A)Cov(A,B)

To find the minimum for any function, we find the derivative and set the derivative equal to zero. Finding the derivative of the variance function with respect to the weight of Asset A, setting the derivative equal to zero, and solving for the weight of Asset A, we find:

*X*A = [σ – Cov(A,B)]/[σ + σ– 2Cov(A,B)]

Using this expression, we find the weight of Asset A must be:

*X*A = (.772 – .001)/[.412 + .772 – 2(.001)]

*X*A = .7798

This means the weight of Stock B is:

*X*B = 1 – *X*A

*X*B = 1 – .7798

*X*B = .2202

*b.* Using the weights calculated in part *a*, the expected return of the portfolio is:

E(*R*P) = *X*AE(*R*A) + *X*BE(*R*B)

E(*R*P) = .7798(.10) + .2202(.16)

E(*R*P) = .1132, or 11.32%

*c.* Using the derivative from part *a*, with the new covariance, the weight of each stock in the minimum variance portfolio is:

*X*A = [σ – Cov(A,B)]/[σ + σ– 2Cov(A,B)]

*X*A = (.772 + .05)/[.412 + .772 – 2(–.05)]

*X*A = .7467

This implies the weight of Stock B is:

*X*B = 1 – *X*A

*X*B = 1 – .7467

*X*B = .2533

*d.* The variance of the portfolio with the weights in part *c* is:

σ = *X*σ + *X*σ + 2*X*A*X*BCov(A,B)

σ = (.7467)2(.41)2 + (.2533)2(.77)2 + 2(.7467)(.2533)(–.05)

σ = .1129

And the standard deviation of the portfolio is:

σP = .11291/2

σP = .3359, or 33.59%